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# CAC(Q) CERS FOR SOLID ROCKET MOTORS

In this paper the application of the CAC(Q) technique to Solid Rocket Motors is described. Three equations having different cost drivers are derived, all with good fit statistics. Techniques for selecting among these three "good" equations are also described and derivation of  $T_1$  is demonstrated.

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CAC(Q) CERs FOR SOLID ROCKET MOTORS  
EXCERPTS FROM  
CR-0617  
COST ESTIMATING  
SOLID ROCKET MOTORS  
WITH THRUST VECTOR CONTROL

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## ABSTRACT

CAC(Q) CERs estimate the Cumulative Average Cost at total production quantity Q. The CAC(Q) equation form is given by

$$CAC(Q) = a(Q)^b f(X)$$

where Q is total quantity and  $f(X)$  is a function of physical and/or performance characteristics. At first glance the equation simply looks like a learning curve. However, the coefficient b will capture the learning curve effect and any other quantity related effects, such as degree of automation. Values for b are in the -.2 to -.4 range which is less than a 90% learning curve typical of Solid Rocket Motors.

The logic behind the CAC(Q) equation is that the best data is the total constant year cost and the total quantity procured. Any other data, even individual lot buys, will have anomalies. To attempt to build CERs with lot data or derive theoretical first unit costs ( $T_1$ s), introduces noise into the data that masks the true relationships. This is especially true when learning analysis on individual data points is very noisy, (e.g., derived learning curves with slopes greater than one, poor learning curve fits, etc.).

In this paper the application of the CAC(Q) technique to Solid Rocket Motors is described. Three equations having different cost drivers were derived, all with good fit statistics. Techniques for selecting among these three "good" equations are also described and derivation of  $T_1$  is demonstrated.

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## **PREFACE**

This paper contains excerpts from "Cost Estimating Solid Rocket Motors with Thrust Vector Control", CR-0617, Tecolote Research, Inc., February 1993. This is referred to as Reference 3. Proprietary Data has been removed so that it could be presented at the DoD Cost Analysis Symposium. The paper has also been shortened by removing some of the conversions contained in the original paper. The resulting paper focuses on the development of CAC(Q) CERs, selecting between them, and deriving a  $T_1$  cost.

# 1

## INTRODUCTION

### 1.1 PURPOSE

Tecolote Research has been investigating solid rocket motor costs for the United States Army Space and Strategic Defense Command (USASSDC) Cost Analysis Office (CAO). In a previous study we developed Cost Estimating Relationships (CERs) for rocket propulsion recurring manufacturing costs. This effort was documented in CR-0540, October 1991 (Ref. 1).

The CERs reported in Ref. 1, while significant in the statistical sense, left much to be desired. First, the data set was very small. It consisted of 13 motors from eight DoD procurements. The focus on recurring manufacturing costs meant that we had to use CCDR data. Only 13 data points could be found that had CCDRs.

Furthermore, the data set appeared to have two strata. Seven motors had Thrust Vector Controls (TVC) and the remaining motors were part of missiles that had aerodynamic controls, most often actuator-driven fins. The TVC motors tended to be larger and performed strategic missions, while the aerodynamic missile motors were smaller with tactical missions. In addition, the control costs for the aerodynamic systems could be separated from the motor costs, as the fins and the actuators were located in the aft section of the missile. On the other hand, part of the TVC controls are an integral part of the nozzle, and that part of the control cost could not be separated from the motor costs. Hence, the data set contained strategic motors with partial control costs and tactical motors without control costs.

It was not surprising, therefore, that attempts to develop CERs that estimated the Theoretical first unit cost (T1) were not successful. The differences in definition between tactical and strategic motors added to the variation inherent in many T1 CER developments. However, we were successful in developing equations that estimated the cumulative average costs at total buy quantity, "Q." These equations are referred to as CAC(Q) equations.

The logic behind CAC(Q) CERs is that the best data is the total constant year cost and the total quantity procured. Any other data, even individual lot buys, will have anomalies.

So to attempt to build CERs with lot data, or derived T1s, introduces noise into the data that masks the true relationships.

The CAC(Q) equation form is shown below. At first glance, it simply looks like a learning curve, with  $f(X)$  representing some function of physical and performance characteristics  $X$  that are hypothesized to explain the cost. However, the coefficient  $b$  on the total production quantity  $Q$  will include not only the learning curve effect, but also any other effect that is associated with quantity. The best example of another quantity-related effect is the degree of automation. The manufacturer will automate a production line more when a large total production quantity is expected. For these large production buys, the average cost at total production quantity is less than it would be if automation were the same for all data points in the data set. This, in turn, leads to a value for the coefficient  $b$  that is much less than that for a typical learning curve. Values in the -.2 to -.4 range are common. While part of the coefficient value represents learning (-.152 for a 90% learning curve), the rest is due to automation or some other quantity-related cause.

$$CAC(Q) = a(Q)^b f(X)$$

The practical consequence of CERs with this form is that you cannot directly calculate a T1. Putting a 1 in the equation for the value of  $Q$  is not an estimate of T1. It is only an estimate of the T1 cost if you are going to produce only one unit and never any more. In effect, you would have no automation and a much higher cost. As a result, the reader is cautioned as follows: DO NOT ENTER 1 FOR  $Q$  TO ESTIMATE A T1 COST.

The correct way to calculate a T1 cost is first to calculate a cumulative average cost at total production quantity and then to convert that cost to T1 by applying a learning curve with an appropriate slope. An example of this calculation is given below.

Suppose the CAC(Q) equation in thousands of dollars is given by the following, where IT is total impulse in thousands of pound-seconds.

$$CAC(Q) = 61.353(Q)^{-0.5298} (IT)^{0.6607}$$



Further, suppose that you want to estimate the T1 cost of motor X, which has a total impulse of 700K pound-seconds, and that you are going to produce 1000 motors. Finally, assume a 95% learning curve. Then the cumulative average cost for 1000 units is

$$CAC(1000) = 61.353(1000)^{-0.5298} 700^{0.6607} = \$119.7K$$

T1 is then found by

$$CAC(1000) = T1(1000)^{-0.074} \text{ or } T1 = \$119.7K/.6 = \$199.5K$$

The CAC(Q) equation form was applied to the solid rocket motor data set in Ref. 1 with significantly better results than those achieved with the T1 form CERs. For example, the Root Mean Square Error (RMS) for the T1 equations ranged from 72% to 84%. For the CAC(Q) equations the RMS error dropped to around 64% and also identified an outlier. When this outlier was removed, the RMS error dropped to 27%.

However, the coefficient on Q appeared to be too high. Typically, this coefficient was less than -.5. Our concern was that the results may be spurious. We managed to tie strategic and tactical motors, (partially) with and without control costs, into the same data set because all strategic motors had smaller total production quantities than the tactical motors. The fear was that the equation form would give wrong results if used to estimate the cost of high total quantity strategic motors (above 600) or low total quantity tactical motors (below 2000).

To examine this concern, we recommended in Ref. 1 that the data be stratified and separate equations be built for the two strata. If the value of the coefficient b is thereby reduced to the more practical -.2 to -.4 range, then the individual strata equations should be used to project into this middle total production quantity range of more than 600 strategic motors and fewer than 2000 tactical motors.

The purpose of this study was to report on the results of developing CERs for the strategic motors with TVC. The results are significantly better than those achieved with the Ref. 1 data set.

## 1.2 DATA SET SELECTION

The first consideration in this investigation was to pick the data set. Stratifying the Ref. 1 recurring manufacturing cost data set left only seven data points at most, and one of those was suspect. That motor had a recurring manufacturing to recurring production cost ratio that was completely out of the range of the other data points. Its value is 0.396, while the average of the remaining six motors is 0.752 with a range from 0.669 to 0.804. With such a small data set, the results would be tenuous at best.

The data set size could be expanded to seven if we investigated recurring production costs instead of recurring manufacturing costs, as we could add the suspect data point. This wasn't much of an improvement in database size.

However, we have recurring production costs for other motors from previous studies, and we were able to find recurring production costs on 22 strategic motors with TVC. This wider data set offered real potential to build a useful and robust CER. This data set was selected for the study.

## 1.3 ORGANIZATION

The recommended equations are presented in Section 2. They are CAC(Q) equations that estimate the recurring production costs. Three equations are presented along with examples of their use. In Section 3, we examine the three equations and give our advice as to which equation to select. Conclusions are reported in Section 4.

## RECURRING PRODUCTION EQUATIONS

### 2.1 EQUATION FORM

The 22-point data set was used for CER development. All of the data points are strategic motors with TVC. The costs are in thousands of FY88 dollars and include the motor and TVC. They also include Systems Engineering and Program Management (SEPM) costs and, hence, are typical of a subcontractor cost to the prime.

The equation form that we used is

$$CAC(Q) = aQ^b(\text{SIZE})^c(\text{NUMBER NOZZLES})^d e^{D1} f^{D2}$$

where a, b, c, d, e, and f are coefficients to be estimated, Q is the total production quantity to be procured, CAC(Q) is the cumulative average cost in thousands of FY88 dollars, and D1 and D2 are dummy variables for material type, defined by

Material Type	D1	D2
Kevlar (Composite)	1	0
Titanium or Glass	0	1
All Other (Steel)	0	0

The equation form incorporates Total Quantity, Size, Number of Nozzles, and Material Type. Three different size variables were investigated. These are Total Weight, Nozzle Weight, and Total Impulse. All of the size variables produced statistically significant results. These are summarized on the following page:

STATISTICS	
Sample Size	22
Degrees of Freedom	16
RMS error	18.4% - 19.1%
Adjusted R2	91.84% - 92.75%
Coefficient t Statistics	All significant
b values	-0.29 to -0.36

As can be seen, the results are very good for each of the size variables. The problem then becomes one of choosing between three good equations. This will be the subject of Section 3. CERs for each of the size variables are given in Sections 2.2, 2.3, and 2.4 respectively. An example of each equation use is also given.

## 2.2 TOTAL WEIGHT EQUATION

The total weight equation is given by:

$$CAC(Q) = 29.045Q^{-0.3387} TW^{0.5126} NN^{0.6167} (1.6680)^{D1} (1.3867)^{D2}$$

where

CAC(Q)	=	Cumulative Average Cost in thousands of FY88 dollars
Q	=	Total Production Quantity
TW	=	Total Weight in pounds
NN	=	Number of Nozzles
D1	=	Kevlar stratification variable
D2	=	Titanium or Glass stratification variable

The significant statistics for this equation are summarized below:

Data Points	22	R-Squared (Adj)	91.84%
Degrees of Freedom	16	F Statistic	48.30
Standard Error (SE)	0.2267	RMS Error	19.1%

### Coefficient Significance

Variable	Coefficient	t-Statistic	Probability Not Zero
Intercept	a	5.04	1.000
Q	b	-5.41	1.000
TW	c	9.46	1.000
NN	d	6.96	1.000
D1	e	3.21	0.995
D2	f	2.56	0.979

### Data Ranges:

$$\begin{aligned}71 &\leq Q \leq 2249 \\3200 &\leq TW \leq 107000 \\1 &\leq NN \leq 4\end{aligned}$$

One data point, Motor 14, exhibits a 32.6% error and is listed as showing an unusual value in the outlier analysis. All the other data points are estimated within 32%. Percentage errors for each of the data points are given in Section 3.2.

As can be seen from these statistics, the total weight equation is highly significant. Furthermore, the coefficient on Q is in the acceptable range, with a little less than half of the quantity devoted to learning curve slope (-0.152 for a 90% learning curve).

As an example of using the equation, assume that we want to estimate the motor cost of missile X. We are going to produce 1000 motors. The Total Weight is 700 pounds, with a single nozzle weighing 30 pounds. The material to be used is Kevlar. The Total Impulse is 300 thousand pound-seconds.

The cumulative average cost of 1000 motors is given by

$$CAC(Q) = 29.045(1000)^{-0.3387}(700)^{0.5126}(1)^{0.6167}(1.6680)^{D1}(1.3867)^{D2}$$

which, after performing the arithmetic, equals \$134K.

To calculate a T1 cost, one cannot use a Q value equal to one, but rather one must assume a learning curve slope in conjunction with the CAC(Q) results at total production quantity Q. Assuming 90%, we have the following:

$$CAC(1000) = T1(1000)^{-0.152} \text{ or } T1 = 134 \text{ K}/.35 = 383\text{K}$$

### 2.3 NOZZLE WEIGHT EQUATION

The nozzle weight equation is given by

$$CAC(Q) = 97.453Q^{-0.2893}NW^{0.5929}NN^{0.4774}(1.7553)^{D1}(1.2601)^{D2}$$

where

CAC(Q)	=	Cumulative Average Cost in thousands of FY88 dollars
Q	=	Total Production Quantity
NW	=	Nozzle Weight in pounds
NN	=	Number of Nozzles
D1	=	Kevlar stratification variable
D2	=	Titanium or Glass stratification variable

The significant statistics for this equation are summarized below:

Data Points	22	R-Squared (Adj)	92.75%
Degrees of Freedom	16	F Statistic	54.73
Standard Error (SE)	0.2138	RMS Error	18.9%

### Coefficient Significance

Variable	Coefficient	t-Statistic	Probability Not Zero
Intercept	a	8.50	1.000
Q	b	-4.85	1.000
NW	c	10.13	1.000
NN	d	5.42	1.000
D1	e	3.77	0.998
D2	f	1.92	0.927

### Data Ranges:

$$71 \leq Q \leq 2249$$

$$90 \leq NW \leq 1540$$

$$1 \leq NN \leq 4$$

Two data points, Motors 14 and 8, exhibit a 30.8% and 50.1% error, respectively, and are listed as showing an unusual value in the outlier analysis. All the other data points are estimated within 26%. Percentage errors for each of the data points are given in Section 3.2.

As can be seen from these statistics, the nozzle weight equation is highly significant. Furthermore, the coefficient on Q is in the acceptable range, with a little more than half of the quantity effect devoted to learning curve slope (-0.152 for a 90% learning curve).

As an example of using the equation, assume that we want to estimate the motor cost of missile X. We are going to produce 1000 motors. The Total Weight is 700 pounds, with a single nozzle weighing 30 pounds. The material to be used is Kevlar. The Total Impulse is 300 thousand pound-seconds.

The cumulative average cost of 1000 motors is given by:

$$CAC(Q) = 97.453(1000)^{-0.2893}(30)^{0.5929}(1)^{0.4774}(1.7553)^1(1.2601)^0$$

which, after performing the arithmetic equals, \$174K.

To calculate a T1 cost, one cannot use a Q value equal to one, but rather one must assume a learning curve slope in conjunction with the CAC(Q) results at total production quantity Q. Assuming 90%, we have the following:

$$CAC(1000) = T1(1000)^{-0.152} \text{ or } T1 = 174K/.35 = 497K$$

## 2.4 TOTAL IMPULSE EQUATION

The total impulse equation is given by

$$CAC(Q) = 77.595Q^{-0.3597} TI^{0.5081} NN^{0.6116} (1.4433)^{D1} (1.2939)^{D2}$$

where

CAC(Q)	=	Cumulative Average Cost in thousands of FY88 dollars
Q	=	Total Production Quantity
TI	=	Total Impulse in thousands of pound-seconds
NN	=	Number of Nozzles
D1	=	Kevlar stratification variable
D2	=	Titanium or Glass stratification variable

The significant statistics for this equation are summarized below:

Data Points	22	R-Squared (Adj)	92.65%
Degrees of Freedom	16	F Statistic	53.94
Standard Error (SE)	0.2153	RMS Error	18.4%



### Coefficient Significance

Variable	Coefficient	t-Statistic	Probability Not Zero
Intercept	a	7.79	1.000
Q	b	-6.07	1.000
TI	c	10.05	1.000
NN	d	7.26	1.000
D1	e	2.36	0.969
D2	f	2.13	0.951

### Data Ranges:

$$\begin{aligned}71 &\leq Q \leq 2249 \\600 &\leq TI \leq 27000 \\1 &\leq NN \leq 4\end{aligned}$$

No data points show an unusual value in the outlier analysis. All the data points are estimated within 32.52%. Percentage errors for each of the data points are given in Section 3.2.

As can be seen from these statistics, the total impulse equation is highly significant. Furthermore, the coefficient on Q is in the acceptable range, with a little less than half of the quantity effect devoted to learning curve slope (-0.152 for a 90% learning curve).

As an example of using the equation, assume that we want to estimate the motor cost of missile X. We are going to produce 1000 motors. The Total Weight is 700 pounds, with a single nozzle weighing 30 pounds. The material to be used is Kevlar. The Total Impulse is 300 thousand pound-seconds.

The cumulative average cost of 1000 motors is given by:

$$CAC(Q) = 77.595(1000)^{-0.3597}(300)^{0.5081}(1)^{0.6116}(1.4433)^1(1.2939)^0$$

which, after performing the arithmetic, equals \$132K.

To calculate a T1 cost, one cannot use a Q value equal to one, but rather one must assume a learning curve slope in conjunction with the CAC(Q) results at total production quantity Q. Assuming 90%, we have the following:

$$CAC(1000) = T1(1000)^{-0.152} \text{ or } T1 = 132K/.35 = 377K$$

### SELECTING A CER

Three very good equations were presented in Section 2. The only difference in form is the size variable. How does one choose a CER from the equations based on Total Weight, Nozzle Weight, or Total Impulse? In this section, we address this question by examining the traditional statistics (3.1), comparing the fit for individual data points (3.2), and seeing how well the equation estimates smaller motors (3.3).

#### 3.1 TRADITIONAL STATISTICS

A number of statistical measures are presented for each equation in Sections 2.2, 2.3, and 2.4. Information on how well the equations fit, and hence can predict, are summarized in these statistics. We have selected four for comparison. These are shown below.

#### STATISTICS

	Equation Based On		
	Total Weight	Nozzle Weight	Total Impulse
R2 (ADJ)	91.84%	92.75%	92.65%
Standard Error	0.2267	0.2138	0.2153
RMS Error	19.1%	18.9%	18.4%
Number of Outliers	1	2	0

From these statistics, it appears that the Total Impulse equation is marginally better. Most significantly, it has no outliers. Its RMS Error is best, and it has the second-best standard error and Adjusted R2.

However, from a statistical point of view, there is really not much difference between the three equations. The choice, therefore, may depend most on the information available to the cost estimator. Are estimates of all three size variables available, and what confidence is there in their values? For example, Nozzle Weight is often not available as early as the other two.

### 3.2 ANALOG

Another means of choosing is by analogy. In this case, one selects a motor that is most like the one being estimated. The equation that has the smallest percent error for the selected motor is the one that is preferred.

Percent errors for all the motors in the database are given in the table below. Separate entries have been made for each of the three equations. A positive entry in the table means that the equation estimated high. A negative entry means that the equation estimated low. Thus, for example, Motor 1 is estimated low by 8.9% using the total weight equation.

PERCENT ERROR

Motor	Equation Based On		
	Total Weight	Nozzle Weight	Total Impulse
Motor 1	-8.90	-16.46	-8.73
Motor 2	-13.55	-5.37	-7.98
Motor 3	-2.77	-6.43	-2.85
Motor 4	-15.48	-11.19	-16.55
Motor 5	-16.07	-11.38	-10.91
Motor 6	27.87	24.50	24.42
Motor 7	24.92	21.52	23.50
Motor 8	1.97	50.13	-7.40
Motor 9	27.00	-14.88	32.52
Motor 10	12.21	1.96	13.95
Motor 11	-26.44	-25.19	-23.59
Motor 12	1.93	-9.10	1.12
Motor 13	-17.23	-10.42	-14.56
Motor 14	-32.60	-30.78	-29.01
Motor 15	13.85	9.51	11.06
Motor 16	22.50	25.99	20.54
Motor 17	-6.78	1.09	-6.61
Motor 18	8.57	14.64	4.92
Motor 19	31.06	11.43	32.07
Motor 20	-24.23	-8.81	-27.54
Motor 21	18.28	3.48	16.35
Motor 22	14.36	22.94	12.10

If the booster that the analyst needs to estimate is most similar to the Motor 21, then the equation on Nozzle Weight seems to be best. It overestimated the cost by 3.48%.

### 3.3 EXTRAPOLATION ESTIMATING

A third means of choosing between equations arises when the motor to be estimated falls outside or nearly outside the range of the data. This is true for most of the motors being considered for TMD and NMD applications. These motors tend to be at the low end of the database, i.e., they are smaller than most or all of the motors in the data set. The relevant question, then, is how well does the equation estimate when extrapolating to smaller motors.

To test this, we ordered the motors in the database by the size variable. We then dropped the smallest motor from the database and refit the equation with the remaining 21 data points. We then predicted the cost of the smallest motor. In this case, Motor 20 was smallest for Total Weight and Total Impulse. Motor 19 was smallest for Nozzle Weight. The Total Weight equation predicted Motor 20 cost low by 34.6%, while the Total Impulse equation predicted low by 39.4%. The Nozzle Weight equation predicted the Motor 19 cost high by 17.7%.

Note that we refer to these calculations as predictions rather than estimations. This is to denote that there is a prediction being made, as the data point in question is not in the database. This is different from the percent error calculations made in regression analysis, where the estimate is in reality a measurement of how well the equation fit the data point, as the data point was part of the database. In this sense, predictions made from the extrapolation estimating technique are a real estimate of the error that one would encounter.

We repeated the process described above until the sample size remaining reached 11, which still allowed 5 degrees of freedom. At each step, the smallest remaining data point was dropped, the equation refit, and the most recently dropped data point predicted. Although predictions of all the dropped data points could be used, concentrating on the one-step predictions has some statistical advantages. For example, it can be shown under the normal regression theory assumptions that the one-step predictions have statistical properties similar to the database residuals in regression theory. For more about this technique and the statistical properties, see Ref. 2.

Results of this analysis for the 11 smallest data points are summarized in the following table. The percentage error entries in the table represent overestimates if positive, and underestimates if negative. The number in parentheses is the sample size from which the estimate was made.

**PERCENT ERROR IN PREDICTION**  
(Sample Size Used in Prediction)

Motor	Equation Based On		
	Total Weight	Nozzle Weight	Total Impulse
Motor 1	*	*	*
Motor 2	*	*	-13.5 (11)
Motor 3	10.8 (11)	*	*
Motor 4	-23.9 (19)	-11.1 (16)	-22.1 (19)
Motor 5	-22.6 (15)	-3.1 (13)	-17.9 (14)
Motor 6	*	*	*
Motor 7	31.1 (14)	35.6 (12)	29.3 (15)
Motor 8	*	*	*
Motor 9	47.6 (12)	-21.6 (15)	53.6 (12)
Motor 10	*	*	*
Motor 11	*	*	*
Motor 12	*	*	*
Motor 13	*	*	*
Motor 14	*	*	*
Motor 15	*	*	*
Motor 16	*	68.4 (11)	*
Motor 17	-15.1 (20)	3.2 (20)	-18.9 (17)
Motor 18	7.8 (18)	51.3 (14)	-4.1 (20)
Motor 19	36.0 (17)	17.7 (21)	30.5 (18)
Motor 20	-34.6 (21)	-6.9 (19)	-39.4 (21)
Motor 21	31.8 (13)	22.3 (17)	19.3 (13)
Motor 22	14.6 (16)	34.9 (18)	5.5 (16)
Sum	83.5	190.7	22.3
Average	25.5	25.1	23.1
Weighted Average	24.7	22.8	22.8

\*Larger motor not predicted

The sum of the percent errors is an indication of bias. In regression theory, the sum of the residuals is always zero. In extrapolation estimating, this is not the case because the data is never in the database when the regression is performed. The closer the sum of the percent errors is to zero, the less bias. In this case, all three equations tend to over predict, but the Total Impulse equation shows the least bias. The average percent error is calculated on the absolute percent errors. Here all three equations are similar, with Total Impulse performing best. A weighted average is also calculated. Sample size is used for the weight, thus giving greater weight to predictions from the larger databases. Again, all three equations perform similarly, but Total Weight is the worst.

Using extrapolation estimating, it still seems that the Total Impulse equation is the best. It has lower average error and less bias.

Another factor favoring the Total Impulse equation is the stability of the coefficient on the size variable. The coefficient starts at 0.5081. As data is removed, the coefficient gets as low as 0.4788 and as high as 0.5574. There is no real pattern to the variation, and the coefficient value of sample size 11 equation is 0.5034. Contrast this to the coefficient behavior for the Total Weight equation. It starts at 0.5126, gets as high as 0.5721, and finishes for sample size 11 at a low of 0.4921. Worse yet is the Nozzle Weight coefficient, which starts at 0.5929 and gets consistently smaller until it reaches 0.3887 for sample size 11.

## CONCLUSIONS

The investigations reported in this paper were very successful. As stated in Section 1, the results from Ref. 1, though statistically significant, were suspicious. Our concern was that we had tied together two separate populations by using the CAC(Q) equation form and that using the equations to estimate large production strategic motors or small production tactical motors would give misleading results. The solution was to build a motor CER with TVC for the strategic motor population. We hypothesized that if the coefficient on the total production quantity  $Q$  was increased from the  $-.5$  range to the  $-.2, -.4$  range, then the CER would be much more reasonable for cost estimating.

The need for a larger data set forced the study to concentrate on a 22 data point set of recurring production costs instead of a 6 data point set of recurring manufacturing costs. This allowed us to develop three size-based CERs, all with significant statistics and coefficients on  $Q$  in the acceptable range. Furthermore, the statistics for these CERs were much better than those in Ref. 1.

It is our conclusion that the stratification of the data set into motors with TVC was the reason that we obtained these good results. It is our recommendation that these equations be used, instead of those in Ref. 1, to estimate motors with TVC in general and especially when the total production quantity is expected to exceed 600.

The three equations differ only in the size variable. There is an equation based on (1) Total Weight, (2) Nozzle Weight, and (3) Total Impulse. Three ways of selecting the best of these equations are given in Section 3. One of these techniques is based on how well the equation performs when trying to estimate motors smaller than those in the data set. It was shown that the equations have an average error of around 23 to 25 percent when predicting smaller motors outside the data set. In general, the Total Impulse equation seems best. However, the statistical quality of the three equations are very close, and the availability of good input specifications for the size variable may be the most important reason for choosing among the equations.



A final word of caution when using these equations. They are CAC(Q) equations and hence estimate the cumulative average cost at total production quantity. Do not enter one for quantity unless you are estimating a motor with only one production unit. If you do enter one for Q, your estimate will be high whenever more than one unit is produced and/or production tooling has been bought. The proper method to calculate T1 costs was presented. This shows the analyst how to correctly calculate a T1 cost from the CAC(Q) cost equations in Section 2.

## REFERENCES

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